String Breaking in Non-Abelian Gauge Theories with Fundamental Matter Fields

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We present clear numerical evidence for string breaking in three-dimensional SU(2) gauge theory with fundamental bosonic matter through a mixing analysis between Wilson loops and meson operators representing bound states of a static source and a dynamical scalar. The breaking scale is calculated in the continuum limit. In units of the lightest glueball we find $r_\mathrm{b}\, m_G \approx 13.6$. The implications of our results for QCD are discussed.

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It has been a long-standing problem in QCD to detect "string-breaking", i.e. the breakdown of linear confinement, by means of computer simulations. String breaking is expected to occur when the energy of a gauge string between static colour sources separated by a distance r becomes as large as the energy required to produce a light quark-antiquark pair, each of which is subsequently bound to the static colour sources. Thus, for distances r below a typical scale r_b the static quarks support a gauge string, resulting in a linearly rising potential, whereas for $r > r_\mathrm{b}$ the ground state potential is bounded by the energy of two static-light "mesons", $V(r) \simeq E_{MM}$. The usual computational strategy is to extract $V(r)$ from Wilson loops calculated in Monte Carlo simulations and to look for signs that the linear rise in $V(r)$ turns into a constant behaviour as the separation r is increased beyond the breaking scale. Despite recent efforts [1–3], hard evidence for string breaking has not been presented so far.

It has been suggested that the apparent failure to detect string breaking is due to insufficient overlap of Wilson loops with the two-meson state [4]. Indeed, a recent analysis of the problem using strong coupling ideas [5] suggests that string breaking is a mixing phenomenon, involving both the string and the two-meson state. Thus, in order to confirm the mixing picture, the conventionally used Wilson loops have to be supplemented by explicit two-meson operators. However, given the complexity and computational cost of full QCD simulations, it will take a big effort to obtain an unambiguous signal for string breaking, even if mixing is taken into account.

In this letter we study the mixing between the string and a two-"meson" state in a simplified theory, the three-dimensional SU(2) Higgs model. We present clear evidence for string breaking from Monte Carlo simulations and essentially confirm the mixing scenario of ref. [5]. Although the three-dimensional SU(2) Higgs model is usually applied to physical contexts other than the strong interactions, its phase diagram has a confinement region that shares a number of features with QCD, such as a linearly rising potential of static colour charges and its eventual screening through matter pair creation. As in QCD, the screening phenomenon has so far not been observed in simulations employing Wilson loops only [6]. In our previous studies of the model [8–10] we have investigated a possible mechanism how the confining properties of the model are lost in the Higgs region through flux tube decay [9]. Furthermore, an attempt to estimate the screening length by using non-local gauge-invariant operators has been described in [10].

Bosonic matter fields imply an enormous simplification of the computational effort, whilst preserving the underlying mechanism for string breaking to occur. The presence of a weak or even moderately strong scalar self-coupling, which has no analogue in QCD, is not expected to change the dynamics of gauge and scalar fields so dramatically that the interpretation of results in a QCD context is no longer possible [8,9].

The study of the model in three dimensions also offers a number of advantages: first one can simulate relatively large volumes, so that large separations of colour charges can be probed efficiently without suffering from significant finite-volume effects. Second, due to superrenormalisability, the curves of constant physics can be mapped out exactly using two-loop perturbation theory [7]. Furthermore, the bare gauge coupling g^2 is a dimensionful quantity, which sets the scale in three-dimensional gauge theories.

We work with the lattice action defined by

$$
\begin{aligned}
S[U,\phi] \;&= \beta_G \sum_p \left(1 - \tfrac{1}{2}\mathrm{Tr}\, U_p\right) \qquad (1)\\
&+ \sum_x \Big\{ -\beta_H \sum_{\mu=1}^{3} \tfrac{1}{2}\mathrm{Tr}\left(\phi^\dagger(x) U_\mu(x)\phi(x+\hat\mu)\right)\\
&+ \tfrac{1}{2}\mathrm{Tr}\left(\phi^\dagger(x)\phi(x)\right) + \beta_R\left[\tfrac{1}{2}\mathrm{Tr}\left(\phi^\dagger(x)\phi(x)\right) - 1\right]^2 \Big\},
\end{aligned}
$$

where $U_\mu(x) \in \mathrm{SU(2)}$ is the link variable, U_p denotes the plaquette, and $\phi(x)$ is the scalar field. The bare parameters $\beta_G \equiv 4/(ag^2), \beta_H$ and β_R are the inverse gauge coupling, the scalar hopping parameter, and scalar

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self-coupling, respectively. In our Monte Carlo simulations we update the gauge fields using a combination of the standard heatbath and over-relaxation algorithms for SU(2) [11,12]. The scalar fields are updated using the algorithm described in [13]. Further details about our simulation procedure can be found in ref. [8].

The standard operator used to compute the static potential is the Wilson loop, i.e. the correlation of a string of length r over a time interval t. Throughout this work we have computed Wilson loops of area $r \times t$ for "on-axis" orientations in the spatial directions, so that all separations are integer multiples of the lattice spacing a:

$$\begin{aligned} G_{SS}(r,t) \ &= \Big\langle \mathrm{Tr}\Big\{ U(0,r\hat{\jmath})\, U(r\hat{\jmath}, r\hat{\jmath}+t\hat{3}) \\ &\times U^\dagger(t\hat{3}, r\hat{\jmath}+t\hat{3})\, U^\dagger(0,t\hat{3}) \Big\} \Big\rangle, \end{aligned} \tag{2}$$

where $U(x,y)$ is a shorthand notation for the straight line of links connecting the sites x and y. A non-local, gauge-invariant operator describing the bound state of a static colour source and a scalar field in three dimensions has been studied in [10], viz.

$$G_M(t) = \left\langle \tfrac{1}{2}\, \mathrm{Tr}\, \left\{ \phi^\dagger(t\hat{3})\, U^\dagger(0,t\hat{3})\, \phi(0) \right\} \right\rangle . \tag{3}$$

This motivates our definition of an operator which projects onto two of these bound states separated by a distance r

$$\begin{aligned} G_{MM}(r,t) \ &= \Big\langle \tfrac{1}{4}\, \mathrm{Tr} \Big\{ (1-\sigma_3)\phi^\dagger(t\hat{3})\, U^\dagger(0,t\hat{3})\, \phi(0) \Big\} \quad (4) \\ &\times \ \mathrm{Tr}\Big\{ (1-\sigma_3)\phi^\dagger(r\hat{\jmath})\, U(r\hat{\jmath}, r\hat{\jmath}+t\hat{3})\, \phi(r\hat{\jmath}+t\hat{3}) \Big\} \Big\rangle. \end{aligned}$$

Finally, we consider operators which describe transitions between a string and a two-meson state and vice-versa:

$$\begin{aligned} G_{SM}(r,t) \ &= \Big\langle \tfrac{1}{2}\, \mathrm{Tr} \Big\{ \phi^\dagger(t\hat{3})\, U^\dagger(0,t\hat{3})\, U(0,r\hat{\jmath}) \\ &\times U(r\hat{\jmath}, r\hat{\jmath}+t\hat{3})\, \phi(r\hat{\jmath}+t\hat{3}) \Big\} \Big\rangle \end{aligned} \tag{5}$$

$$\begin{aligned} G_{MS}(r,t) \ &= \Big\langle \tfrac{1}{2}\, \mathrm{Tr} \Big\{ \phi^\dagger(r\hat{\jmath})\, U(r\hat{\jmath}, r\hat{\jmath}+t\hat{3}) \\ &\times U^\dagger(t\hat{3}, r\hat{\jmath}+t\hat{3})\, U^\dagger(0,t\hat{3})\, \phi(0) \Big\} \Big\rangle. \end{aligned} \tag{6}$$

In order to study the static potential and the mixing between gauge string and two-meson states, we construct the following matrix correlator

$$G(r,t) = \begin{pmatrix} G_{SS}(r,t) & G_{SM}(r,t) \\ G_{MS}(r,t) & G_{MM}(r,t) \end{pmatrix}. \tag{7}$$

The operator $G_M(t)$ serves to extract the energy E_M of a single bound state of a static source and a scalar field, which helps to identify the two-meson state, since one naively expects $E_{MM} \simeq 2E_M$.

In order to enhance the signal of all our operators, we have constructed smeared gauge and scalar fields. Following the standard algorithm in [14] we constructed "fuzzed" link variables of length a. Fuzzed scalar fields were constructed using a suitable adaptation of the blocking algorithm used in [8–10]. In particular, eq. (3) of ref. [9] carries over literally after replacing the blocked links by their fuzzed counterparts. We constructed spatial Wilson lines for three different link fuzzing levels, whereas five fuzzing levels were used for scalar fields, so that we obtained the correlator $G(r,t)$ as an 8×8 matrix.

A procedure of diagonalising $G(r,t)$ following a variational method has been described in detail in [8,15]. Here we only wish to point out that the diagonalisation of the 8×8 correlation matrix G_{ik} yields eigenvectors Φ_i from which the correlation function of the (approximate) eigenstates of the Hamiltonian can be calculated,

$$\Gamma_i(t) = \langle \Phi_i(t)\Phi_i(0) \rangle = \sum_{j,k=1}^{8} a_{ij} a_{ik}\, G_{jk}(t). \tag{8}$$

The coefficients a_{ik} quantify the overlap of each individual correlator G_{ik} with the correlators of the mass eigenstates, Γ_i. The diagonalisation procedure allows the computation of the ground state energy, the first few excitations, and, most crucially, their composition in terms of the original operator basis. The coefficients a_{ik} play a central role for the interpretation of string breaking as a mixing phenomenon.

Our choice of bare parameters followed closely our earlier work [8–10]. In particular, we approach the continuum limit for a fixed ratio $\beta_G\, \beta_R/\beta_H^2 = 0.0239$, corresponding to a fixed continuum scalar self-coupling. We selected three values for the bare gauge coupling, i.e. $\beta_G = 5.0, 7.0, 9.0$, and the hopping parameter β_H was always chosen such that the model was in the confinement region of the phase diagram. Lattice sizes were chosen such that $L/\beta_G \simeq 5.7$. At $\beta_G = 5.0$ we also considered $L/\beta_G = 7.2$ in order to check for finite-size effects. The maximum separation of colour charges was $r_{\max} = L/2$ in all simulations. At $\beta_G = 5.0$ we used 30 iterations in the fuzzing algorithm, which was sufficient to observe saturation in the projection onto the ground state. The link/staple mixing ratio was always set to two. For the runs at smaller lattice spacing, the maximum number of fuzzing iterations was scaled by the respective ratio of β_G-values. The correlators $G(r,t)$ and $G_M(t)$ were measured after every full update, and 50 individual measurements were collected in bins for post-processing. Typically we performed 1500 measurements (30 bins) and 3000 measurements for our most accurate determinations. Statistical errors were estimated using a jackknife procedure.

In Fig. 1 we show the energies of the two lowest eigenstates as a function of r/a. The ground state shows the familiar linear rise for small separations. For $r/a \geq 15$ however, the linear rise saturates, and a comparison with the energy extracted for the single meson state shows that $E(r) \simeq 2E_M$ in this region. The qualitative and quantitative behaviour of the ground state energy at large

separations is entirely consistent with the string breaking picture in the sense that the ground state energy is bounded by that of the two-meson state. Thus, we take $r_\mathrm{b} \simeq 15a$ as a rough estimate at this point in parameter space. The energy of the first excited state is constant for $r < r_\mathrm{b}$ and consistent with that of a two-meson state. A significant r-dependence in this region is only observed at very small separations. There are a number of possible explanations, such as interactions between the two mesons, mixing with the excited potential, or that the separation may be too small for a two-meson state to exist. By contrast, for $r > r_\mathrm{b}$, the energies of the first excited state rise linearly and appear to be the continuation of the ground state potential below the breaking scale.

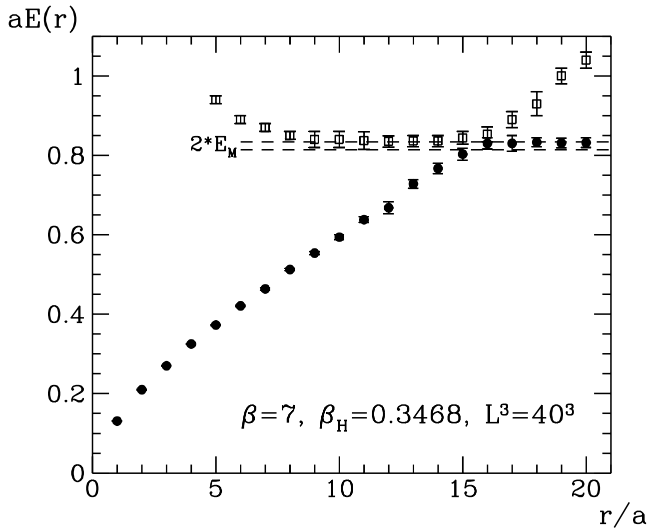

FIG. 1. The energies of the ground state and the first excited state for $\beta_G = 7.0, \beta_H = 0.3468$ on 40^3. The dashed lines indicate the location of twice the energy of the single meson state, as extracted from $G_M(t)$.

The interpretation of the r-dependence of the spectrum as clear evidence for string breaking is corroborated by the analysis of the composition of the energy eigenstates in terms of the original operator basis. In Fig. 2 (a) we plot the maximum projection of the string and two-meson operator onto the ground state. At small r/a it is obvious that the ground state is dominated by the string operator, consistent with the picture of static quarks bound by a flux tube, whereas the projection of the two-meson operator onto the ground state is always significantly lower, but clearly non-vanishing. As mentioned above, the admixture of G_{MM} to the ground state may have physical reasons, but it could also arise through overlapping fuzzed links in G_{MM}, which effectively create a loop of links. Operator artefacts of this kind were already encountered in [9]. Thus, the most likely interpretation of the overlaps at small r/a is a combination of physical effects and operator artefacts. With increasing r/a G_{MM} almost fully projects onto the two-meson state. For $r \simeq r_\mathrm{b}$ however, there are clear indications for mixing between the flux tube and the two-meson state, with comparable projections of both operator types. For $r > r_\mathrm{b}$ the ground state receives almost exclusively contributions from the two-meson operator, and it is now the first excited state (not shown), whose dominant operator content comes from the string. We conclude that there is clear evidence for the crossing of energy levels associated with gauge strings and two-meson operators at r_b, so that the flux tube is energetically disfavoured for $r > r_\mathrm{b}$.

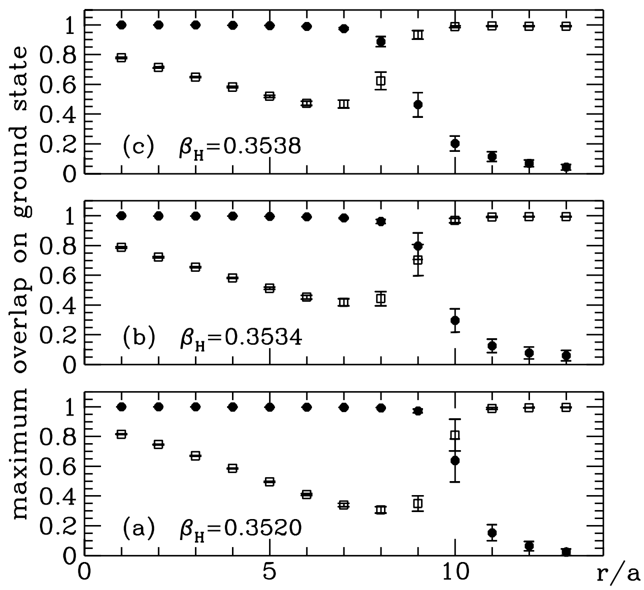

FIG. 2. The maximum projections on the ground state of the string ($a_{1,SS}$, full circles) and the two-meson operator ($a_{1,MM}$, open squares) for several values of the hopping parameter at $\beta_G = 5.0$ on 26^3.

At both $\beta_G = 5.0, \beta_H = 0.3520$ and $\beta_G = 7.0, \beta_H = 0.3468$ we observe a fairly sharp crossover and thus a relatively narrow mixing region. In order to explore the mixing phenomenon for smaller scalar masses, we have simulated a sequence of β_H-values at $\beta_G = 5.0$ fixed, which all satisfy the constraint $\beta_G\, \beta_R/\beta_H^2 = 0.0239$, i.e. correspond to the same continuum scalar coupling. For smaller scalar mass (larger β_H) one expects r_b to occur at smaller separations, since less energy is required to support a two-meson state. Also, according to [5] the width of the mixing region becomes larger for smaller scalar mass, so that the crossover is expected to widen as β_H is increased. Our plots of the maximum projections as a function of β_H displayed in Fig. 2 (a)–(c) show that both is indeed the case. The conclusions of [5], which have been obtained for SU(N) gauge theories with fermions, are thus confirmed qualitatively in our simplified model.

By comparing the spectrum for different β_H at $\beta_G = 5.0$ fixed, we observe that energies extracted from eigenstates with dominant admixtures of gauge strings remain largely unaffected by the value of β_H, and only the en-

ergies of states with a dominant overlap from two-meson operators show a significant dependence on β_H. This is in line with our earlier observation that scalar and gauge sectors decouple approximately [8,9]. The consequence of this in the present context is that the string operator predominantly projects on the state with unbroken flux tube for *all* spatial separations outside the mixing region. Hence, Wilson loops exhibit an area law for correlation distances t/a up to which the signal can be followed, even if r/a is already beyond $r_{\rm b}$. This offers a possible explanation for the failure to detect string breaking in full QCD simulations using only Wilson loops. Namely, since the only significant overlap of a string onto the broken flux tube occurs inside the mixing region and vanishes again when r/a is increased further, the saturation of the energy can only be observed if the mixing region is broader than, say, two lattice spacings. Furthermore, the width of the mixing region is sensitive to the mass of the matter considered. Therefore, in computations of Wilson loops in full QCD with relatively heavy dynamical quarks, the mixing region may be too narrow. If this scenario holds, then the problem can only be overcome by employing a variational approach of the type discussed in this letter.

We now consider the scaling behaviour of the breaking scale $r_{\rm b}$. As a definition of $r_{\rm b}$ we use the point where the maximum overlaps of string and two-meson operators onto the ground state are equal, i.e.

$$\Delta \equiv a_{1,SS} - a_{1,MM} = 0, \tag{9}$$

where we have borrowed from the notation in eqs. (2), (4) and (8). An estimate of $r_{\rm b}/a$ is obtained through a local, linear interpolation of Δ to the point where it vanishes. This procedure becomes exact in the continuum limit. Within our accuracy we assign an error of at most one lattice spacing to the estimate of $r_{\rm b}/a$. Our results for all lattice sizes and parameter values are shown in Table I. We note again that $r_{\rm b}/a$ decreases as the scalar mass is decreased. Furthermore, by comparing the estimates for $r_{\rm b}/a$ on $L/a = 26$ and 36 at $\beta_G = 5.0$ we conclude that finite-size effects are practically absent in our simulations. In the last column $r_{\rm b}$ is given in units of the continuum gauge coupling constant g^2. Within errors we observe scaling of the estimates for $r_{\rm b}\,g^2$ from runs c, e and f, whose bare parameters are related through lines of constant physics in the continuum. For a controlled continuum extrapolation, a further value at larger β_G would be desirable, so we choose to quote as our final result

$$r_{\rm b}\,g^2 \approx 8.5. \tag{10}$$

This number can be combined with results for the mass spectrum in the continuum limit at the same physical couplings [8]. Taking $m_S/g^2 = 0.839(15)$ for the lightest scalar bound state and $m_G/g^2 = 1.60(4)$ for the lightest scalar glueball [8], we obtain

$$r_{\rm b}\,m_G \approx 13.6,\;\; m_S/m_G = 0.524(16) \tag{11}$$

To summarise, our results clearly show that string breaking occurs in non-Abelian gauge theories with matter fields. The main evidence is the existence of a mixing region, where both the string and two-meson operators have a significant overlap onto the ground state. We have also shown that Wilson loops are not suitable to compute the ground state potential for $r > r_{\rm b}$. Estimates of $r_{\rm b}$ are also of phenomenological interest. For instance, $r_{\rm b}$ is expected to be related to the scale parameter of QCD through light-quark constituent masses [16]. Also, $r_{\rm b}$ is important for the understanding and description of fragmentation processes [17]. An extension of our analysis to QCD is therefore highly desirable.

TABLE I. Estimates for the breaking scale $r_{\rm b}$.

Run	β_G	β_H	L/a	$r_{\rm b}/a$	$r_{\rm b}\,g^2$
a	5.0	0.3538	26	8.2 ± 0.5	6.56 ± 0.38
b	5.0	0.3534	26	8.9 ± 1.0	7.12 ± 0.79
c	5.0	0.3520	26	9.8 ± 0.7	7.88 ± 0.59
d	5.0	0.3520	36	9.9 ± 0.8	7.90 ± 0.64
e	7.0	0.3468	40	14.8 ± 1.0	8.47 ± 0.57
f	9.0	0.3438	52	19.1 ± 1.0	8.50 ± 0.44

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